INDEX OF REFRACTION - PRISM SPECTROMETER

PURPOSE
The objectives of this experiment are
• to calibrate a prism spectrometer with a standard (mercury) light source
• to measure the index of refraction of a glass prism for several wavelengths of light

THEORY
The instrument you will be using in this laboratory is called a prism spectrometer. The prism spectrometer is an instrument designed to separate the wavelengths of a light source into its various spectral components and to measure these wavelengths. A schematic overhead view of a spectrometer is given in figure 1 below. A short description of the spectrometer follows.

Light from a source enters the spectrometer through a thin vertical slit. The width of this slit can be adjusted to vary the amount of light entering the spectrometer. The light then passes through a collimator which produces a parallel beam of light rays. Exiting the collimator the light encounters a prism. The prism is the chief component in the spectrometer; it separates the light into its spectral components by refracting each wavelength through a different angle. The angular position of each spectral component can then be located with a telescope. This telescope is free to move on a circular scale, which is calibrated for measuring the angles through which the telescope or the prism table is rotated.

Figure 2 shows a ray of light of a single wavelength, $\lambda$, incident upon an equilateral prism. Let the index of refraction of the glass at this particular wavelength be $n$. It is important to remember that the index of refraction depends on the wavelength $\lambda$. It is not a constant! We shall take the index of refraction of the surrounding air to be 1.0. Strictly speaking, this is not correct but our spectrometer is not precise enough to detect the difference. Let the apex angle of the prism be denoted by $A$. Since our prism is equilateral, the apex angle $A$ of the prism can be taken as 60°, although its value can be determined accurately in the laboratory using the prism spectrometer.
We can apply Snell's Law to the ray of light at each surface. This leads to the two equations,

\[ \sin \theta_1 = n \sin \theta_2 \quad (1a) \]
\[ n \sin \theta_3 = \sin \theta_4 \quad (1b) \]

The angles \( \theta_2 \) and \( \theta_3 \) are not independent, being related by the equation,

\[ A = \theta_2 + \theta_3 \quad (2) \]

You are asked to prove this relation in your pre-lab. The angle \( D \), known as the angle of deviation, is shown in Figure 2. \( D \) is the angle between the incident ray and the emergent ray. This angle is directly measurable in the laboratory. \( D \) is the exterior angle to the small triangle formed by the dashed lines and the ray that passes through the prism. The exterior angle theorem from geometry says that the exterior angle to any triangle is equal to the sum of the two opposite interior angles of the triangle. These interior angles in our triangle are the angles \((\theta_1 - \theta_2)\) and \((\theta_4 - \theta_3)\).

Applying the exterior angle theorem we get,

\[ D = (\theta_1 - \theta_2) + (\theta_4 - \theta_3) \quad (3) \]

Combining this equation with Equation 2, we have

\[ D = \theta_1 + \theta_4 - A \quad (4) \]

For reasons to be discussed below, we would like to express \( D \) in terms of the single angle \( \theta_2 \). This is accomplished by using Equations 1a and 1b. Solving these equations for \( \theta_1 \) and \( \theta_4 \), respectively, we get

\[ \theta_1 = \sin^{-1}(n \sin \theta_2) \quad (5a) \]
\[ \theta_1 = \sin^{-1}(n \sin(A - \theta_2)) \quad (5b) \]
where we have used the relation \( \theta_3 = A - \theta_2 \) from Equation 2 in Equation 5b. Finally, substituting these two equations into Equation 4 we get,

\[
D = \sin^{-1}(n \sin \theta_2) + \sin^{-1}(n \sin(A - \theta_2)) - A \tag{6}
\]

There is a good reason for expressing the angle D in terms of the single angle \( \theta_2 \). For a certain angle \( \theta_2 \), D possesses a minimum value. To show this, one differentiates Equation 6 with respect to \( \theta_2 \) and sets the result equal to zero. This produces an equation for the angle \( \theta_2 \). The solution of this equation is,

\[
\theta_2 = A/2. \tag{7}
\]

You are asked to carry out this procedure and prove Equation 7 in the Prelab. Now, from Equation 2 we see that when \( \theta_2 = A/2 \), \( \theta_3 = A/2 \) also. This means that the angle D has its minimum value \( D_m \) when the incident ray is directed in such a way that the light passes through the prism symmetrically with respect to apex A. The angle \( D_m \) is known as the *minimum angle of deviation* for the prism at the wavelength \( \lambda \). It is this particular value of D which we will measure in the laboratory. The experimental technique for finding \( D_m \) is given in the Procedure. Knowing \( D_m \) for a particular wavelength, we can find the index of refraction of the material of the prism for that wavelength, as follows. Note that since \( \theta_2 = \theta_3 \) when D is at its minimum value, we see from Equations 1a and 1b (and from the symmetry of the problem) that \( \theta_1 = \theta_4 \) as well. Then from Equation 4 we have,

\[
\theta_4 = (D_m + A)/2 \tag{8}
\]

Substituting this into Equation 1b and solving for the index of refraction gives,

\[
n = \frac{\sin((D_m + A)/2)}{\sin(A/2)} \tag{9}
\]

Our source of light is a mercury gas discharge tube that acts in much the same way as a familiar fluorescent lamp. To the naked eye the light emitted by the mercury tube appears to be nearly white. However, viewed through a spectrometer, the light is separated into distinct spectral components or *lines* (see the color plates in your textbook.). Each spectral line corresponds to a single wavelength of light. Since the index of refraction is dependent on the wavelength, each wavelength will be refracted through a different angle. With the spectrometer we can measure the minimum angle of deviation \( D_m \) for each wavelength. Then using Equation 9 we can calculate the index of refraction for each wavelength. A description of the lines you will measure and their corresponding wavelengths is given in Figure 3 below. From your data and the wavelengths below you can construct a plot of the index of refraction, n, vs. wavelength, \( \lambda \).
With the development of quantum theory in the earlier part of this century, the lines within the spectrum of gases such as mercury became understood as a manifestation of the quantization of energy. The spectrometer was instrumental in the testing of this theory. You may already know that each element has a unique spectrum that can serve as a fingerprint for the element. By measuring the wavelengths that are present within an arbitrary spectrum we can identify the elements which are present in the source of the light, be it a gas or other light emitting object. Indeed, this is our only recourse in identifying the constituents of a far away star or galaxy. In order to identify the wavelengths that are present within a spectrum, the spectrometer must be calibrated with a known standard such as mercury. This calibration is carried out by plotting the wavelength $\lambda$ vs the minimum angle $D_m$ for all the observable lines in the spectrum of the standard. Such a plot will have a downward sloping trend similar to that shown in Figure 4.

If an unknown spectrum is observed, the minimum angles of deviation for the spectral lines can be measured and by using the calibration curve the wavelengths which correspond to the observed minimum angles can be determined.
EXPERIMENTAL PROCEDURE

1. Adjustments of the spectrometer for parallel light

The three basic parts of a spectrometer are (1) the telescope - to be focused to receive parallel rays, (2) the prism table mounted on a rotating circular table having two vernier scales and (3) the collimator - to render incident rays from the Hg lamp parallel.

Focusing the eyepiece on the cross-hairs

Pull the eyepiece almost out of the telescope by the ring and insert it slowly until the cross hairs are in focus with the eyes relaxed. In order to avoid eye fatigue the eyepiece should be in focus after the eye has been focused for distance vision.

Adjustments of telescope to receive parallel rays

Turn the telescope towards a distant object (e.g. a far off building) and adjust the telescope to see a clear well-defined image of the distant object. If the initial adjustment of the cross hairs was good, the image of the distant object will remain fixed against the image of cross hairs as the eye is moved slightly from side to side. That is, there is no parallax between the images of the cross hairs and the distant object, and thus the two coincide with the focal plane of the telescope. Otherwise there is relative motion between the image of the cross hairs and that of the distant object and the adjustments must be repeated. This method of focusing by the elimination of parallax is universal in the use of optical instruments.

Adjustments of the collimator to provide parallel incident rays

Swing the telescope so that it is aligned with the collimator. There are two knobs under the telescope. One of these is to fix the telescope relative to the collimator and the other for fine alignment of the telescope with respect to the collimator. Note that there is another set of knobs for the prism table. One of them is to fix the prism table at any position and the other is used for finer adjustments. The slit side of the collimator should face the light source. Illuminate the collimator slit with the mercury light source. Make the slit width narrow and bring the slit into sharp focus while viewing through the eyepiece side of the telescope and by adjusting the collimator only. Again use the absence of parallax between the slit image and the cross hairs.

After these adjustments have been made, care must be taken that they are not disturbed during the entire experiment.

2. Determination of the precision of angular measurement in the spectrometer

The circular main scale on the spectrometer is graduated from zero to 360 degrees; each division on this scale is equal to one half of a degree (i.e. 30 minutes). There are 30 divisions on each of the vernier scales (1) and (2). When the zero of either of the vernier scales matches with a division on the main scale, the 30th division on that vernier coincides with another main scale division. Careful observation indicates that there are 29 main scale divisions between these zero and 30th divisions of the vernier scale. This implies that each vernier division corresponds to (29/30) of a main scale division and that the angular measurements can be made to a precision of
one minute (i.e., the difference between the values of one main scale division and one vernier scale division). Remember that one minute equals 1/60 th of a degree.

3. Determination of the angle of minimum deviation

(a) Place the prism on the table in a position similar to that shown in Figure 2, so that a spectrum - with yellow deviated the least and blue deviated the most - is formed by the prism.

Locate this spectrum first with your eye and then swing the telescope into position to pick up the spectrum.

Rotate the circular table - note that the prism also rotates - so that the spectrum moves toward the straight-through position.

Follow the motion of the spectrum with the telescope. It will be noted that at one point the motion of the telescope will have to be reversed, though the prism continues to move in the same direction; that is, the deviation reaches a minimum and then increases. This turning position corresponds to that of minimum deviation.

Locate this position accurately. Set the cross hair on the yellow line of the mercury spectrum and fix the prism table and the telescope. If the prism table is rotated now, the yellow line approaches and coincides with the crosshairs, and moves back. Be sure to show this adjustment to the instructor.

Fix the prism table with its knob.

Read the prism table angular position as 'a' with a magnifying glass. In general, the zero for the vernier division is between two main scale divisions. Note the main scale reading to the left of the zero of the vernier. Find the number (n) of the vernier division that coincides with a main scale division. Thus

"α" = main scale reading + n(1 minute).

Enter "α " in Table 1

(b) Remove the prism and swing the telescope, without disturbing the prism table, so that it is sighting directly down the collimator.

Fix the telescope and use the fine adjustment knob to set the cross hairs coincident with the image of the slit and read the straight through angle β. Note that you have to read both the main and vernier scales.

Enter the angle "β" in Table 1.

(c) Repeat steps (a) and (b) for green, blue and violet lines of the spectrum. Note that the direct reading has to be taken in each case and that this reading need not remain the same.
Enter your readings in Table 1.

4. The refractive index of the material of the prism for each of the wavelengths corresponding to yellow, green, blue and violet will be determined separately using Equation 9.

5. A calibration curve of wavelengths of these lines vs. the angles of minimum deviation should be constructed.
TABLE 1: Angle of minimum deviation measurements

<table>
<thead>
<tr>
<th>Color/Wavelength</th>
<th>Minimum Deviation, $\alpha$</th>
<th>Direct Reading, $\beta$</th>
<th>$D_m = (\alpha - \beta)$</th>
<th>$n$</th>
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<tbody>
<tr>
<td>Yellow 579 nm</td>
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<tr>
<td>Green 546 nm</td>
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<td>Blue 436 nm</td>
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<tr>
<td>Violet 408 nm</td>
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